Linear Equations Reference Sheet

Equation of a Line

Gradient-Intercept Form

$$y = mx + b$$

m – gradient (slope)
b – y-intercept

Useful for interpreting the relationship between variables x and y, graphing, and identifying m and b. Also useful for generating the equation of the line when m and b are given (ie from a graph).

Point-Gradient Form

$$y - y_1 = m(x - x_1)$$

(x_1, y_1) – a point on the line
m – gradient (slope)

Useful for plugging in a point on the line (once the gradient is known) to generate the equation of the line. It may then be changed into another form.

Standard Form

$$Ax + By = C$$

A is non-negative and A, B, C are relatively prime integers (no common factors)

Useful for finding both the x- and y-intercepts of the line and using the intercepts to graph the line.

General Form

$$Ax + By + C = 0$$

A is non-negative and A, B, C are relatively prime integers (no common factors)

Similar to standard form except it is solved for 0 (all terms on left side). Not very useful for knowing anything about the line without formulas. Recommend converting to another form.

Horizontal Line

$$y = b$$

All points have y-coordinate b

The line is horizontal with gradient $m = 0$ and y-intercept b.

Vertical Line

$$x = a$$

All points have x-coordinate a

The line is vertical with an undefined gradient.

Gradient (Slope)

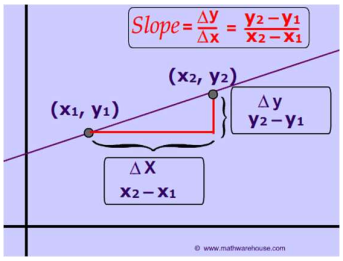
Gradient Formula

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

(x_1, y_1) & (x_2, y_2) – points on the line

The formula is "rise over run", which is the change in y divided by the change in x.

Reading Gradient From a Graph



It is recommended to read the gradient from left to right. If the line goes up from left to right, the gradient is positive. If the line goes down from left to right, the gradient is negative.

Parallel Lines

$$m_1 = m_2$$

The gradients of parallel lines are equal. Parallel lines never intersect.

Perpendicular Lines

$$m_1 = -\frac{1}{m_2}$$

The gradients of perpendicular lines are negative reciprocals of each other. Perpendicular lines intersect at 90° (right) angles.

Important Formulas

Mid-Point Formula

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

(x_1, y_1) & (x_2, y_2) – coordinates

The formula can be broken up into two parts, the x-part and y-part. Each part is the middle (or average) of the x and y coordinates of the given points.

Distance Formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

(x_1, y_1) & (x_2, y_2) – coordinates

The formula is derived from the Pythagorean Theorem $a^2 + b^2 = c^2$. The legs of a right triangle are given by the difference in x- and y-coordinates (inside the brackets). The distance is the hypotenuse of this right triangle. This is calculated by the square root of the sum of the squares of the legs (right-hand-side of formula).

Perpendicular Distance

$$d = \frac{|Ax_1 + Bx_2 + C|}{\sqrt{A^2 + B^2}}$$

Given an equation in the form $Ax + By + C = 0$ and a point (x_1, y_1)

The equation of the line must be in the above form and the perpendicular (shortest) distance from the point to the line can be calculated by plugging in all values.