# Algebra Cheat Sheet

## Basic Properties & Facts

### Arithmetic Operations

$ab+ac=a(b+c)$ $\qquad a\left(\frac{b}{c}\right)=\frac{ab}{c}$

$\frac{\left(\frac{a}{b}\right)}{c}=\frac{a}{bc}$ $\qquad \frac{a}{\left(\frac{b}{c}\right)}=\frac{ac}{b}$

$\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$ $\qquad \frac{a}{b}-\frac{c}{d}=\frac{ad-bc}{bd}$

$\frac{a-b}{c-d}=\frac{b-a}{d-c}$ $\qquad \frac{a+b}{c}=\frac{a}{c}+\frac{b}{c}$

$\frac{ab+ac}{a}=b+c,\ a\neq 0$ $\qquad \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)}=\frac{ad}{bc}$

Denominator values $\neq 0$

### Exponent Properties

$a^n a^m = a^{n+m}$ $\qquad \frac{a^n}{a^m}=a^{n-m}=\frac{1}{a^{m-n}}$

$\left(a^n\right)^m = a^{nm}$ $\qquad a^0=1,\ a\neq 0$

$(ab)^n = a^n b^n$ $\qquad \left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}$

$a^{-n}=\frac{1}{a^n}$ $\qquad \frac{1}{a^{-n}}=a^n$

$\left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^n=\frac{b^n}{a^n}$ $\qquad a^{\frac{n}{m}}=\left(a^{\frac{1}{m}}\right)^n=\left(a^n\right)^{\frac{1}{m}}$

### Properties of Radicals

$\sqrt[n]{a}=a^{\frac{1}{n}}$ $\qquad \sqrt[n]{ab}=\sqrt[n]{a}\sqrt[n]{b}$

$\sqrt[m]{\sqrt[n]{a}}=\sqrt[nm]{a}$ $\qquad \sqrt[n]{\frac{a}{b}}=\frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

$\sqrt[n]{a^n}=a$, if $n$ is odd

$\sqrt[n]{a^n}=|a|$, if $n$ is even

### Properties of Inequalities

If $a<b$ then $a+c<b+c$ and $a-c<b-c$

If $a<b$ and $c>0$ then $ac<bc$ and $\frac{a}{c}<\frac{b}{c}$

If $a<b$ and $c<0$ then $ac>bc$ and $\frac{a}{c}>\frac{b}{c}$

### Properties of Absolute Value

$$|a|=\begin{cases} a & \text{if } a\geq 0 \\ -a & \text{if } a<0 \end{cases}$$

$|a|\geq 0$ $\qquad |-a|=|a|$

$|ab|=|a||b|$ $\qquad \left|\frac{a}{b}\right|=\frac{|a|}{|b|}$

$|a+b|\leq|a|+|b|$ Triangle Inequality

### Distance Formula

If $P_1=(x_1,y_1)$ and $P_2=(x_2,y_2)$ are two points the distance between them is

$$d(P_1,P_2)=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

### Complex Numbers

$i=\sqrt{-1}$ $\qquad i^2=-1$ $\qquad \sqrt{-a}=i\sqrt{a},\ a\geq 0$

$(a+bi)+(c+di)=a+c+(b+d)i$

$(a+bi)-(c+di)=a-c+(b-d)i$

$(a+bi)(c+di)=ac-bd+(ad+bc)i$

$(a+bi)(a-bi)=a^2+b^2$

$|a+bi|=\sqrt{a^2+b^2}$ Complex Modulus

$\overline{(a+bi)}=a-bi$ Complex Conjugate

$\overline{(a+bi)}(a+bi)=|a+bi|^2$

**Logarithms and Log Properties**

*Definition*

$y = \log_b x$ is equivalent to $x = b^y$

*Example*

$\log_5 125 = 3$ because $5^3 = 125$

*Special Logarithms*

$\ln x = \log_e x$ natural log

$\log x = \log_{10} x$ common log

where $e = 2.718281828\ldots$

*Logarithm Properties*

$\log_b b = 1 \qquad \log_b 1 = 0$

$\log_b b^x = x \qquad b^{\log_b x} = x$

$\log_b \left(x^r\right) = r \log_b x$

$\log_b (xy) = \log_b x + \log_b y$

$\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$

The domain of $\log_b x$ is $x > 0$

## Factoring and Solving

**Factoring Formulas**

$x^2 - a^2 = (x+a)(x-a)$

$x^2 + 2ax + a^2 = (x+a)^2$

$x^2 - 2ax + a^2 = (x-a)^2$

$x^2 + (a+b)x + ab = (x+a)(x+b)$

$x^3 + 3ax^2 + 3a^2x + a^3 = (x+a)^3$

$x^3 - 3ax^2 + 3a^2x - a^3 = (x-a)^3$

$x^3 + a^3 = (x+a)\left(x^2 - ax + a^2\right)$

$x^3 - a^3 = (x-a)\left(x^2 + ax + a^2\right)$

$x^{2n} - a^{2n} = \left(x^n - a^n\right)\left(x^n + a^n\right)$

If $n$ is odd then,

$x^n - a^n = (x-a)\left(x^{n-1} + ax^{n-2} + \cdots + a^{n-1}\right)$

$x^n + a^n$

$= (x+a)\left(x^{n-1} - ax^{n-2} + a^2x^{n-3} - \cdots + a^{n-1}\right)$

**Quadratic Formula**

Solve $ax^2 + bx + c = 0$, $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $b^2 - 4ac > 0$ - Two real unequal solns.

If $b^2 - 4ac = 0$ - Repeated real solution.

If $b^2 - 4ac < 0$ - Two complex solutions.

**Square Root Property**

If $x^2 = p$ then $x = \pm\sqrt{p}$

**Absolute Value Equations/Inequalities**

If $b$ is a positive number

$|p| = b \quad \Rightarrow \quad p = -b$ or $p = b$

$|p| < b \quad \Rightarrow \quad -b < p < b$

$|p| > b \quad \Rightarrow \quad p < -b$ or $p > b$

**Completing the Square**

Solve $2x^2 - 6x - 10 = 0$

(1) Divide by the coefficient of the $x^2$

$$x^2 - 3x - 5 = 0$$

(2) Move the constant to the other side.

$$x^2 - 3x = 5$$

(3) Take half the coefficient of $x$, square it and add it to both sides

$$x^2 - 3x + \left(-\frac{3}{2}\right)^2 = 5 + \left(-\frac{3}{2}\right)^2 = 5 + \frac{9}{4} = \frac{29}{4}$$

(4) Factor the left side

$$\left(x - \frac{3}{2}\right)^2 = \frac{29}{4}$$

(5) Use Square Root Property

$$x - \frac{3}{2} = \pm\sqrt{\frac{29}{4}} = \pm\frac{\sqrt{29}}{2}$$

(6) Solve for $x$

$$x = \frac{3}{2} \pm \frac{\sqrt{29}}{2}$$

# Functions and Graphs

**Constant Function**

$y = a$ or $f(x) = a$

Graph is a horizontal line passing through the point $(0, a)$.

**Line/Linear Function**

$y = mx + b$ or $f(x) = mx + b$

Graph is a line with point $(0, b)$ and slope $m$.

*Slope*

Slope of the line containing the two points $(x_1, y_1)$ and $(x_2, y_2)$ is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$

*Slope – intercept form*

The equation of the line with slope $m$ and $y$-intercept $(0, b)$ is

$$y = mx + b$$

*Point – Slope form*

The equation of the line with slope $m$ and passing through the point $(x_1, y_1)$ is

$$y = y_1 + m(x - x_1)$$

**Parabola/Quadratic Function**

$y = a(x-h)^2 + k \quad f(x) = a(x-h)^2 + k$

The graph is a parabola that opens up if $a > 0$ or down if $a < 0$ and has a vertex at $(h, k)$.

**Parabola/Quadratic Function**

$y = ax^2 + bx + c \quad f(x) = ax^2 + bx + c$

The graph is a parabola that opens up if $a > 0$ or down if $a < 0$ and has a vertex at $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

**Parabola/Quadratic Function**

$x = ay^2 + by + c \quad g(y) = ay^2 + by + c$

The graph is a parabola that opens right if $a > 0$ or left if $a < 0$ and has a vertex at $\left(g\left(-\frac{b}{2a}\right), -\frac{b}{2a}\right)$.

**Circle**

$(x-h)^2 + (y-k)^2 = r^2$

Graph is a circle with radius $r$ and center $(h, k)$.

**Ellipse**

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Graph is an ellipse with center $(h, k)$ with vertices $a$ units right/left from the center and vertices $b$ units up/down from the center.

**Hyperbola**

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Graph is a hyperbola that opens left and right, has a center at $(h, k)$, vertices $a$ units left/right of center and asymptotes that pass through center with slope $\pm\frac{b}{a}$.

**Hyperbola**

$$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$$

Graph is a hyperbola that opens up and down, has a center at $(h, k)$, vertices $b$ units up/down from the center and asymptotes that pass through center with slope $\pm\frac{b}{a}$.

# Common Algebraic Errors

| Error | Reason/Correct/Justification/Example |
|---|---|
| $\frac{2}{0} \neq 0$ and $\frac{2}{0} \neq 2$ | Division by zero is undefined! |
| $-3^2 \neq 9$ | $-3^2 = -9$, $(-3)^2 = 9$ Watch parenthesis! |
| $\left(x^2\right)^3 \neq x^5$ | $\left(x^2\right)^3 = x^2x^2x^2 = x^6$ |
| $\frac{a}{b+c} \neq \frac{a}{b} + \frac{a}{c}$ | $\frac{1}{2} = \frac{1}{1+1} \neq \frac{1}{1} + \frac{1}{1} = 2$ |
| $\frac{1}{x^2+x^3} \neq x^{-2} + x^{-3}$ | A more complex version of the previous error. |
| $\frac{\cancel{a}+bx}{\cancel{a}} \neq 1+bx$ | $\frac{a+bx}{a} = \frac{a}{a} + \frac{bx}{a} = 1 + \frac{bx}{a}$<br>Beware of incorrect canceling! |
| $-a(x-1) \neq -ax-a$ | $-a(x-1) = -ax+a$<br>Make sure you distribute the "-"! |
| $(x+a)^2 \neq x^2+a^2$ | $(x+a)^2 = (x+a)(x+a) = x^2+2ax+a^2$ |
| $\sqrt{x^2+a^2} \neq x+a$ | $5 = \sqrt{25} = \sqrt{3^2+4^2} \neq \sqrt{3^2} + \sqrt{4^2} = 3+4 = 7$ |
| $\sqrt{x+a} \neq \sqrt{x} + \sqrt{a}$ | See previous error. |
| $(x+a)^n \neq x^n+a^n$ and $\sqrt[n]{x+a} \neq \sqrt[n]{x} + \sqrt[n]{a}$ | More general versions of previous three errors. |
| $2(x+1)^2 \neq (2x+2)^2$ | $2(x+1)^2 = 2\left(x^2+2x+1\right) = 2x^2+4x+2$<br>$(2x+2)^2 = 4x^2+8x+4$<br>Square first then distribute! |
| $(2x+2)^2 \neq 2(x+1)^2$ | See the previous example. You can not factor out a constant if there is a power on the parethesis! |
| $\sqrt{-x^2+a^2} \neq -\sqrt{x^2+a^2}$ | $\sqrt{-x^2+a^2} = \left(-x^2+a^2\right)^{\frac{1}{2}}$<br>Now see the previous error. |
| $\frac{a}{\left(\frac{b}{c}\right)} \neq \frac{ab}{c}$ | $\frac{a}{\left(\frac{b}{c}\right)} = \frac{\left(\frac{a}{1}\right)}{\left(\frac{b}{c}\right)} = \left(\frac{a}{1}\right)\left(\frac{c}{b}\right) = \frac{ac}{b}$ |
| $\frac{\left(\frac{a}{b}\right)}{c} \neq \frac{ac}{b}$ | $\frac{\left(\frac{a}{b}\right)}{c} = \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{1}\right)} = \left(\frac{a}{b}\right)\left(\frac{1}{c}\right) = \frac{a}{bc}$ |

 

| Properties of Equality | Property | Example(s) |
|---|---|---|
| Addition Property of Equality | If a = b and c = d, then a + c = b + d | If x = 2 and y = 3, then x + y = 5<br>If x = 3 and 2 = 2, then x + 2 = 5 |
| Subtraction Property of Equality | If a = b and c = d, then a - c = b - d | If x = 5 and y = 12, then y - x = 7<br>If x = 14 and 12 = 12, then x - 12 = 2 |
| Multiplication Property of Equality | If a = b and c = d, then ac = bd | If x = 8 and y = 9, then xy = 72<br>If x = 2 and 5 = 5, then 5x = 10 |
| Division Property of Equality | If a = b and c = d ? 0, then a/c = b/d and a/b = c/d | If x = 6 and y = 2, then x/y = 3 and x/6 = y/2<br>If 5x = 15, and 5 = 5, then x = 3 |
| Reflexive Property of Equality | a = a | 15 = 15<br>x = x |
| Symmetric Property of Equality | If a = b, then b = a | If x = 4, then 4 = x<br>If y = 16, then 16 = y |
| Transitive Property of Equality | If a = b and b = c, then a = c | If x = y and y = 12, then x = 12<br>If x = 7 and 7 = y, then x = y |
| Substitution Property | If a = b, then either can be substituted for the other | If x = y + 2, and y = 6, then x = 8<br>If 2x + xy = 35, and x = 7, then 2(7) + 7y = 35 |
| Zero Product Property | If ab = 0, then a = 0 or b = 0 (or both) | If xy = 0, then x = 0 or y = 0<br>If 8x = 0, then 8 = 0 or x = 0 (thus x = 0)<br>If (5-y)(y) = 0, then 5 - y = 0 or y = 0 |

FYI... Parentheses (), brackets [], and Braces {} form groups of number and these groups act like single numbers.

| Property # | Name | Example |
|---|---|---|
| 1 | Commutative property of Addition | a + b = b + a<br>a + b + c = b + a + c<br>[(x+y)/2]+b+c=c+b+[(x+y)/2] |
| 2 | Commutative property of multiplication | a * b = b * a<br>a * b * c = b * a * c<br>[(x*y)/2]*b*c=c*b*[(x*y)/2] |
| 3 | Associative Property of Addition | (a + b) + c = a + (b + c)<br>([(x+y)/2]+b)+c=[(x+y)/2] +(b+c) |
| 4 | Associative Property of Multiplication | (a * b) * c = a * (b * c)<br>([(x+y)/2]*b)*c=[(x+y)/2] *(b*c) |
| 5 | Distributive Property | a *( b + c) = (b + c)*a = ab+ac<br><br>[(x+y)/2]*(b+c)= (b+c) *[(x+y)/2] = [(x+y)/2]*b+ [(x+y)/2]*c |
| 6 | Additive Identity (there is a number 0 such that) | a + 0 = 0 + a = a<br><br>[(x+y)/2] + 0 = 0 + [(x+y)/2] = [(x+y)/2] |
| 7 | Multiplicative Identity (there is a number 1 such that) | a * 1 = 1 * a = a<br><br>[(x+y)/2] * 1 = 1 * [(x+y)/2] = [(x+y)/2] |
| 8 | Additive Inverse (there is a number such that) | a + (-a) = (-a) + a = 0<br><br>[(x+y)/2] + (-[(x+y)/2]) = (-[(x+y)/2])+ [(x+y)/2] =0 |
| 9 | Multiplicative Inverse (there is a number such that) | $a * \left(\frac{1}{a}\right) = \left(\frac{1}{a}\right) * a = 1$<br>$\left(\frac{(x+y)}{2}\right) * \frac{1}{\frac{(x+y)}{2}} = \frac{1}{\frac{(x+y)}{2}} * \left(\frac{(x+y)}{2}\right) = 1$ provided (x+y)/2 ≠0 |
| 10 | Zero Property | a * 0 = 0 * a =0<br><br>[(x+y)/2] * 0 = 0 * [(x+y)/2] = 0 |

# Table of Properties

Let $a$, $b$, and $c$ be real numbers, variables, or algebraic expressions.
(These are properties you need to know.)

| | Property | Example |
|---|---|---|
| 1. | Commutative Property of Addition<br>$a + b = b + a$ | $2 + 3 = 3 + 2$ |
| 2. | Commutative Property of Multiplication<br>$a \cdot b = b \cdot a$ | $2 \cdot (3) = 3 \cdot (2)$ |
| 3. | Associative Property of Addition<br>$a + (b + c) = (a + b) + c$ | $2 + (3 + 4) = (2 + 3) + 4$ |
| 4. | Associative Property of Multiplication<br>$a \cdot (b \cdot c) = (a \cdot b) \cdot c$ | $2 \cdot (3 \cdot 4) = (2 \cdot 3) \cdot 4$ |
| 5. | Distributive Property<br>$a \cdot (b + c) = a \cdot b + a \cdot c$ | $2 \cdot (3 + 4) = 2 \cdot 3 + 2 \cdot 4$ |
| 6. | Additive Identity Property<br>$a + 0 = a$ | $3 + 0 = 3$ |
| 7. | Multiplicative Identity Property<br>$a \cdot 1 = a$ | $3 \cdot 1 = 3$ |
| 8. | Additive Inverse Property<br>$a + (-a) = 0$ | $3 + (-3) = 0$ |
| 9. | Multiplicative Inverse Property<br>$a \cdot \left(\frac{1}{a}\right) = 1$<br>Note: $a$ cannot $= 0$ | $3 \cdot \left(\frac{1}{3}\right) = 1$ |
| 10. | Zero Property<br>$a \cdot 0 = 0$ | $5 \cdot 0 = 0$ |

Remember the order of Mathematical operations PEMDAS…

| | | |
|---|---|---|
| P | Parentheses | Perform all operations inside parentheses first |
| E | Exponentiation | Then perform any Exponential operations next |
| M | Multiplication | Follow this up with any multiplication |
| D | Division | Next do the division |
| A | Addition | Addition follows |
| S | Subtraction | and finally do the subtraction |